QUALITATIVE REALIZATION OF SECOND ORDER DIGITAL DIFFERENTIATOR

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ABSTRACT

This paper presents the basic design of stable, second order low pass IIR digital differentiator. This design is obtained by inverting the response of second order integrator obtained by interpolating Simpson and trapezoidal rules. The resulting transfer function is stabilized and the magnitude is compensated. Because of the low order and high accuracy these filters are preferred.

Keywords: IIR Filter, Low pass filter, differentiator

1. INTRODUCTION

Differentiators are basic building block of low pass filter and can be extensively used in various signal processing fields such as digital control [1], communication [2] and biomedical applications [3]. In many applications, differentiation is followed by low-pass filtering. Differentiation is used to abstract data about swift transients in the signal. Low-pass filters are used to reject noise frequencies greater than the cut-off frequencies of the signal. Low-pass filtering and differentiation are realized as a single low-pass differentiator filter or by cascading low-pass filter and a differentiating filter. The resulting low-pass differentiators are called IIR differentiators.

In this paper the design of low-pass differentiators using low-order IIR filters is carried out. Here differentiation is followed by low-pass filtering [4], [5]; Differentiation is used to extract information about sharp transients in the signal while lowpass filtering is used to reject high frequency noises [4]. Firstly a digital integrator [5] is designed and then inverts the transform to obtain the response of analog differentiator [6] [7]. In the next step the response of differentiator is stabilized and then later on discretises the response(s to z transformation) [8] [9]. Finally the response is checked by passing a sinusoidal signal inclusive of noise through it

2. INTEGRATOR DESIGN

The basic concept comes from the fact that the response of an ideal integrator lies between the response of rectangular and triangular integrators [10] [11] as follows:

\[
H(z) = a H_{\text{Rect}}(z) + (1-a) H_{\text{Trapezoidal}}(z)
\]

where ‘a’ defines the range between the two integrators and is a real value parameter ranging in closed interval [0 1]. \(H_{\text{Rect}}(z)\) and \(H_{\text{Trapezoidal}}(z)\) are the transfer functions of the Simpsons and the Trapezoidal integrators, respectively, and are shown below:

\[
H_{\text{Rect}}(z) = \frac{\frac{s}{z^2+4z+1}}{3(z^2+1)}
\]

\[
H_{\text{Trapezoidal}}(z) = \frac{7(z+1)}{2(z-1)}
\]

Substituting equations (2) and (3) in (1) and simplifying yields the following transfer function for the novel class of non-minimum phase integrators:

\[
H(z) = \frac{7(3-a)[z^2+\frac{10+12a}{3}z+1]}{6(z^2-1)}
\]

The numerator of (7) is factorized and it gives

\[
H(z) = \frac{7(3-a)(z+r_1)(z+r_2)}{6(z^2-1)}
\]

where \(r_1 = \frac{1+6+2\sqrt{3}}{3-a}\) and \(r_2 = \frac{1+6-2\sqrt{3}}{3-a}\)

This class of integrators has the property that its zeros are reciprocal pairs around the unit circle in the z-plane, since \(r_1 = 1/r_2\) [12].

3. INVERTING RESPONSE TO OBTAIN DIFFERENTIATOR DESIGN AND STABILIZING METHOD

Now, from the family of digital integrators obtained above, new integer-order digital differentiators are obtained. Direct inversion of \(H(z)\) will give an unstable filter as \(H(z)\) has a zero \(r_1\). By reflecting the \(r_1\) to \(1/r_1\), i.e. \(r_2\), \(H(z)\) becomes

\[
H(z) = \frac{K7(3-a)(2+r_2)}{6(z^2-1)}
\]

To determine \(K\), we assume final value of the impulse responses of \(H(z)\) and \(\hat{H}(z)\) be the same, i.e.,
\lim_{z \to 1} (z-1)H(z) = \lim_{z \to 1} (z-1)\mathcal{H}(z)
which gives \(K = r_1\) [7] [9]. So, a new family of the digital differentiators can be given by

\[
w_2(z) = \frac{1}{\mathcal{H}(z)} = \frac{6(z^2-1)}{r_17(3-a)(z+r_2)^2} = \frac{r_16z^2-1}{7(3-a)(z+r_2)^2}
\]

(8)

In this approach the transfer function of an integrator is first inverted, and then the resulting transfer function is stabilized by reflecting the poles that lie outside the unit circle in the \(z\)-plane to inside the unit circle. After that the magnitude is compensated appropriately. In compensation technique pole lying at a radius \(r\) is replaced by a pole that lies at a radius of \(1/r\) and the magnitude of the resulting transfer function is multiplied by \(r\). Thus to compensate for the change in magnitude, the resulting transfer function is multiplied by \(1/r\). The magnitude of the frequency response of the resulting differentiator has the same range and accuracy as the integrator.

4. SIMULATION RESULTS

The pole zero plot obtained in figure 1 shows the unstability of the response obtained from inverting equation 3. The figure 1 clearly shows that the given system is unstable because one of the pole lies outside the unit circle. The pole zero plot of figure 1 is obtained at \(a = 0.4\).

Now by stabilizing approach the above response is stabilized and now it is seen that all the poles lie inside the unit circle making the system stable. This is done by reflecting \(r_1\) to \(1/r_1\) and by compensating the magnitude appropriately. In compensation technique the magnitude response is multiplied by \(r\) if pole lying at radius \(r\) is changed to \(1/r\). The pole zero plot of figure 2 is obtained at \(a = 0.4\).

![Fig. 1: Pole zero response obtained at \(a = 0.4\) (unstable filter)](image1)

![Fig. 2: pole zero response obtained at \(a = 0.4\) (stable filter)](image2)

Figure 3 shows the magnitude response of the stable system obtained in equation 8 at values of ‘\(a\)’ lying between 0 to 1 with 0.2 increment. The new differentiator could have higher accuracy or greater range of frequency than the traditional differentiators.

Figure 4 shows the phase response of the stable system obtained in (8) at values of ‘\(a\)’ lying between 0 to 1 with 0.2 increment. The plot shows that the resulting phase responses are almost linear over the pass bands.

![Fig. 3: Magnitude response (stable filter)](image3)

![Fig. 4: Phase response (stable filter)](image4)

Fig 5(a) shows the input sinusoidal signal, Fig 5(b) shows the sinusoidal signal with noise and Fig 5(c) shows the final filtered signal. The filtered signal is obtained after passing the distorted input signal through IIR low pass differentiator filter. The IIR differentiator removes the noise and inserts the phase shift of 90 degree which can be verified as:

In fig 6(a) the sin wave cycle start from \(x=0\) and maxima occur at \(x=1.5\). Now in the filtered signal shown in fig. 6(c) sin wave cycle start from \(x=1.5\) and maxima occur at \(x=3\) which shows the phase shift of 90 degree.
CONCLUSION
Firstly second-order integrators are designed by interpolating Simpson and Trapezoidal rule. After that the response is inverted to obtain a second-order differentiator, stabilizing the unstabilized response and then compensating the magnitude. The improvement of stabilization is to return minimum phase filters when applied to analog filters having all poles. The designed second order differentiator has desired magnitude response of zero at zero frequency and at Nyquist frequency. The phases of designed differentiator are almost linear over desired frequency range. These differentiators can be preferred for real time applications because of their lower order and higher accuracy.

REFERENCES